

Chiral magnetic effect and Maxwell-Chern-Simons electrodynamics in Weyl semimetals

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The Weyl semimetal, due to a non-zero energy difference in the pair of Weyl nodes shows chiral magnetic effect(CME), that is, flow of a dissipationless electric current along an applied magnetic field. Such a chiral magnetic effect in Weyl semimetals has been studied using the laws of classical electrodynamics. It has been shown that the CME in a Weyl semimetal changes the properties namely, frequency dependent skin depth, capacitive transport, plasma frequency etc. in an unexpected way as compared to the conventional metals. This is due to the emergence of a natural length scale in the system called the chiral magnetic length. Such a new observation might help in exploiting these class of materials in the applications in future technology.

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In recent times, there has been an explosion of research in topologically non-trivial states of matter following the remarkable discovery of topological insulators and superconductors[1–8]. The topological order manifested in these systems is not associated with the spontaneously breaking of a symmetry rather can be described by topological invariants. Usually the robust topological protection is associated with a non-zero spectral gap in the bulk and the existence of protected zero energy surface states is regarded as the hallmark of a non-trivial topological phase of matter. However, recently there has been proposal that systems in three spatial dimensions in presence of broken time reversal(TR) symmetry/space inversion(SI) symmetry can also be topologically protected even without bulk energy gap[9–17]. These are called Weyl semimetals(WSM)[9]. Realization of such phenomena has been predicted in pyrochlore iridites[9] and since then, it has been reported to be experimentally confirmed in materials such as, TaP, NbP, TaAs and NbAs[18–23].

A WSM is a three-dimensional analogue of graphene and the low energy excitations with broken TR symmetry can be described by a pair of linearly dispersing massless Dirac fermions governed by the Hamiltonian, $H_\chi(k) = \chi \hbar v_F \vec{k} \cdot \vec{\sigma} - \mu_\chi$, where $\chi = \pm$ is the chirality, v_F , the Fermi velocity, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ refers to three Pauli matrices and μ_χ stands for chirality dependent chemical potential(given by a superposition of the equilibrium carrier density and the pumped carrier density originating from chiral anomaly). Thus, here H_χ describes the two Weyl fermions of opposite chirality. Due to the fermion doubling theorem[24], Weyl nodes with opposite chirality always appear in pairs. The two band touching Weyl points act as a source and a sink(monopole and anti-monopole charges) of Berry curvature which is similar to a fictitious magnetic field on the electron wave function in momentum space[25]. The Weyl nodes can be separated by a wave vector Q in the first Brillouin zone or by an energy offset $\hbar Q_0$. The topological properties of a Weyl semimetal are manifested in the form of a θ -term

contribution to the action $S_\theta = \frac{\alpha}{4\pi^2} \int dt \int d^3r \theta(\vec{r}, t) \vec{E} \cdot \vec{B}$ where $\theta(\vec{r}, t) = 2(\vec{Q} \cdot \vec{r} - Q_0 t)$ is so called axion angle and $\alpha = \frac{e^2}{\hbar c} \approx 1/137$ is the fine structure constant. Here, e is the electron charge and \vec{E}, \vec{B} are respectively the electric and magnetic fields. If the bands are degenerate with $\vec{Q} = 0 = Q_0$, the system does not possess topological properties, that is, such effect would disappear if the two nodes with opposite monopole charges merge and annihilate each other. Due to the non-trivial topology in the momentum space(see Fig. 1(a)), such materials exhibit wide variety of unusual electromagnetic responses[26, 27]. In a Weyl semimetal, electrons near each Weyl node can be associated with a chirality by the monopole charge of that node. In the application of a pair of non-orthogonal electric and magnetic fields, the charges can be transported between the two Weyl nodes with opposite chiralities. Thus, the number of electrons with a definite chirality is no longer conserved implying the so called chiral anomaly[28] in these systems. Such anomaly is predicted to give an enhanced negative magnetoresistance[29] when the applied electric and magnetic fields are parallel to each other. Such predictions have been confirmed by experiments in the material TaAs[30, 31].

In addition, these nodal materials might show chiral magnetic effect(CME) when the energies of the pair of Weyl nodes are different[13, 14, 32–34](see Fig. 1(b)). This effect provides a dissipationless electric current \vec{J}_{ch} flowing along an applied magnetic field \vec{B} ($\vec{J}_{ch} = \sigma_{ch} \vec{B}$) in contrast to the conventional Ohm's law ($\vec{J} = \sigma \vec{E}$). The CME conductivity σ_{ch} , in a low energy effective theory is shown to be proportional to the energy separation between the pair of Weyl nodes[27]. However, at present there is no direct observation of CME in its original sense, that is, an electric current driven by the external magnetic field. Moreover, CME vanishes as a static effect in the thermal equilibrium[35, 36] whereas it might exist in the non-equilibrium limit[32, 37]. Since CME can be regarded as a Chern-Simons extension of

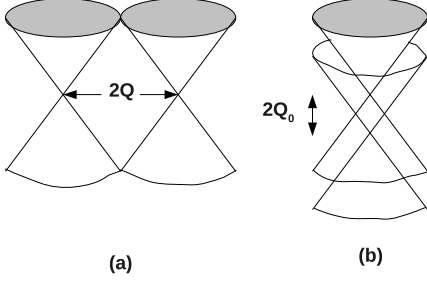


FIG. 1. Schematic diagram of two Weyl nodes in a semimetal separated in (a) momentum ($2\vec{Q}$) and (b) energy ($2Q_0$).

electromagnetism[38] which includes electric and magnetic fields, electric charge and current densities, such phenomena need investigation within the framework of the laws of electrodynamics. This might lead to non-trivial consequences in many of their properties.

In this letter, we start with Maxwell-Chern-Simons equations of electrodynamics and show the effect of CME in the properties such as, frequency dependent skin depth, capacitive transport, plasma frequency etc. in a Weyl semimetal. Further, these properties which look unconventional are compared to that of conventional metals. Such non-trivial observations can be exploited for their applications in future electronics.

The axion term modifies the standard Maxwell equations of electrodynamics in a Weyl semimetal. Thus, the MCS equations can be written as,

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon \quad (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\vec{\nabla} \times \vec{B} = \mu \vec{J} + \mu \vec{J}_{ch} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}, \quad (4)$$

where the source terms respectively are the charge density ρ and the current densities \vec{J} and \vec{J}_{ch} , \vec{J} being the conventional Ohmic current density whereas \vec{J}_{ch} is the CME current density. Here, μ and ϵ respectively are the magnetic permeability and the dielectric permittivity of the material. Since both the current densities are related to conductivities already mentioned in the introduction, the equations containing the source terms (Eq.(1) and (4)) get modified[39]. The charge density in Eq.(1) can be dropped out due to the disappearance of charge accumulation. Due to the coupling of \vec{E} and \vec{B} in the above equations, one can go for higher partial derivatives to decouple them. Thus, the Faraday's and the Ampere's law (Eq.(3) and (4)) yields,

$$\nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu \sigma_{ch} \frac{\partial \vec{B}}{\partial t} \quad (5)$$

$$\nabla^2 \vec{B} = \mu \sigma \frac{\partial \vec{B}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} - \mu^2 \sigma_{ch}^2 \vec{B} - \mu^2 \sigma \sigma_{ch} \vec{E} - \mu^2 \epsilon \sigma_{ch} \frac{\partial \vec{E}}{\partial t}. \quad (6)$$

Using the Fourier mode expansion of both the fields, $\vec{E}(\vec{r}, t) = \vec{E}_0 \exp[-i(\vec{k} \cdot \vec{r} - \omega t)]$ and $\vec{B}(\vec{r}, t) = \vec{B}_0 \exp[-i(\vec{k} \cdot \vec{r} - \omega t)]$, Eqs.(5) and (6) can be simplified as,

$$(k^2 + i\mu\sigma\omega - \mu\epsilon\omega^2)\vec{E}_0 = -i\mu\sigma_{ch}\omega\vec{B}_0 \quad (7)$$

$$(k^2 + i\mu\sigma\omega - \mu\epsilon\omega^2 - \mu^2\sigma_{ch}^2)\vec{B}_0 = (\mu^2\sigma\sigma_{ch} + i\mu^2\epsilon\sigma_{ch}\omega)\vec{E}_0. \quad (8)$$

Thus, the ratio between the amplitudes of both the fields can be written as,

$$\begin{aligned} \frac{B_0}{E_0} &= \frac{(k^2 + i\mu\sigma\omega - \mu\epsilon\omega^2)}{-i\mu\sigma_{ch}\omega} \\ &= \frac{(\mu^2\sigma\sigma_{ch} + i\mu^2\epsilon\sigma_{ch}\omega)}{(k^2 + i\mu\sigma\omega - \mu\epsilon\omega^2 - \mu^2\sigma_{ch}^2)}. \end{aligned} \quad (9)$$

Such an equation yields the dispersion relation (the relation between k and ω) in the form of an algebraic equation as,

$$(k^2 + i\mu\sigma\omega - \mu\epsilon\omega^2)^2 = (k\mu\sigma_{ch})^2. \quad (10)$$

This equation contains four roots, given as,

$$k = \frac{1}{2}[\pm\mu\sigma_{ch} \pm \sqrt{\mu^2\sigma_{ch}^2 - 4(i\mu\sigma\omega - \mu\epsilon\omega^2)}]. \quad (11)$$

Defining a small dimensionless parameter as, $\delta = \frac{\sigma\omega}{\mu\sigma_{ch}^2} \ll 1$ and assuming a good conductor limit, that is, $\frac{\epsilon\omega}{\sigma} \ll 1$, the above dispersion (for positive sign only) can be approximated as,

$$k \simeq \begin{cases} \mu\sigma_{ch}[1 + \delta\frac{\epsilon\omega}{\sigma} - i\delta] \\ i\mu\sigma_{ch}\delta[1 + i\frac{\epsilon\omega}{\sigma}] \end{cases} \quad (12)$$

From these equations, it is obvious that k is in general complex which has real and imaginary parts. Writing $k = k_R \pm ik_I$, one obtains, $k_R = \mu\sigma_{ch}[1 + \delta\frac{\epsilon\omega}{\sigma}] \simeq \mu\sigma_{ch}$ and $k_I = \mu\sigma_{ch}\delta$. Since the imaginary part of k results in the attenuation of the electromagnetic wave, the so called *skin depth* can be calculated to be, $d = \frac{1}{k_I} = (\mu\sigma_{ch}\delta)^{-1}$. Defining a natural characteristic length scale in the system as the *chiral magnetic length* as, $l_{ch} = \frac{\pi}{\mu\sigma_{ch}}$, the skin

depth can be rewritten as, $d = \frac{l_{ch}}{\pi\delta}$. Thus l_{ch} gives a measure of how far the wave penetrates into the Weyl semimetal. It is obvious from this expression that the skin depth in a Weyl semimetal is dependent on frequency through the parameter δ which is quite different from a conventional metal[40]. Considering the parameters from a lattice model[41], that is, taking the energy difference between the Weyl nodes to be $\Delta \sim 100[meV]$, $\sigma_{ch} \sim 10^7 \Delta[A/m^2T] = 10^9[A/m^2T]$, the perturbative parameter δ comes out to be $\delta = 10^{-1}$ for a frequency $\omega = 10^5[Hz]$ and $\frac{\epsilon\omega}{\sigma} = 10^{-12} \ll 1$. The validity of these approximations become better and better for frequency less than $10^5[Hz]$. Since $l_{ch} \sim 10^{-3}$, the skin depth in such a case is estimated as, $d \sim 10^{-2}[m]$ whereas, for a normal metal $d \sim 10^{-3}[m]$. It is obvious that the skin depth in a Weyl semimetal is one order of magnitude larger as compared to that of conventional metal. Here, the parameters considered are, $\sigma \sim 10^6[S/m]$, $\mu = \mu_0 = 10^{-6}[N/A^2]$, $\epsilon = \epsilon_0 = 10^{-11}[C^2/N.m^2]$. The real part of k , on the contrary, determines the propagation speed, wavelength and the index of refraction respectively as, $v = \frac{\omega}{k_R}$, $\lambda = \frac{2\pi}{k_R}$ and $n = \frac{ck_R}{\omega}$.

Putting the expression of k from Eq.(12) in Eq.(9), one obtains,

$$\frac{B_0}{E_0} \approx \frac{i\mu\sigma_{ch}}{\omega} \approx \frac{i\pi}{\omega l_{ch}}. \quad (13)$$

It is obvious from this expression that the natural length l_{ch} characterizes the relation between both the fields E_0 and B_0 . Writing down the wave number as well as both the fields in terms of their magnitudes and phases as, $k = k \exp[i\phi]$, $B_0 = B_0 \exp[i\phi_B]$ and $E_0 = E_0 \exp[i\phi_E]$, one can obtain the relation between the phases of both the fields as, $\phi_B - \phi_E \approx \phi \approx \pi/2$. This implies that the magnetic and the electric fields are no longer in phase, magnetic field lags behind the electric field by a phase of $\pi/2$. This can be understood in a way that the current in such a system leads the voltage by the phase of $\pi/2$, implying that the response is purely capacitive. This result is due to purely CME effect. This is in accordance with a recent prediction on capacitive transport[42] in a WSM. Also, under certain conditions, this might lead to material independent universal effective capacitance in a cylinder geometry[42]. Further, Eq.(13) also justifies the perturbative parameter δ (for $\omega \leq 10^5[Hz]$) which is the ratio between both the fields with their respective conductivities, that is, $\delta \approx |\frac{\sigma E_0}{\sigma_{ch} B_0}| = |\frac{i\sigma\omega}{\mu\sigma_{ch}}|$. This is nothing but the ratio between the ordinary current to that of the CME one and the analysis physically corresponds to a large CME expansion. Since the operational frequency here is in the standard broadcast/radio frequency range, these materials might be applicable in radio electronics.

Next, let us consider the behaviour of the refractive index in a WSM which can provide us a crude estimate of the plasma frequency in such systems. Since the relation between both the fields are known from Eq.(13), the dispersion relation can approximately be written as,

$$k^2 \simeq \mu\epsilon\omega^2 - i\mu\sigma\omega + \mu^2\sigma_{ch}^2. \quad (14)$$

Such a relation can be made meaningful by employing the Drude relation for the AC conductivity, that is, $\sigma(\omega) = \frac{\sigma_0}{1+i\tau\omega}$, ($\sigma_0 = \frac{ne^2\tau}{m}$ for conventional metals but in undoped WSM, $\sigma_0 = \frac{\mu_0^2 e^2 \tau}{12\pi^2 \hbar^3 v_F}$ and in doped WSM $\sigma_0 = \frac{e^2 \tau}{24\pi^2 \hbar^3 v_F}(\mu_{0+}^2 + \mu_{0-}^2)$, μ_0 being the chemical potential whereas $\mu_{0\pm}$ are the chiral chemical potential in WSM)[43]). Considering the collisionless limit $\omega\tau \gg 1$, the conductivity in a conventional metal becomes, $\sigma(\omega) = \frac{ne^2}{im\omega}$ whereas taking the same limit in a WSM, one obtains,

$$k^2 = \mu\epsilon\omega^2[1 - \frac{\omega_p^2}{\omega^2}], \quad (15)$$

where ω_p is the plasma frequency, expressed as, $\omega_p^2 = \frac{1}{\epsilon}(\frac{\mu_0^2 e^2}{12\pi^2 \hbar^3 v_F} - \mu\sigma_{ch}^2) = \frac{1}{\epsilon}[\frac{e^2}{24\pi^2 \hbar^3 v_F}(\mu_{0+}^2 + \mu_{0-}^2) - \mu\sigma_{ch}^2]$. It is already known that for $\omega > \omega_p$, the wave number is real and the wave propagates without any attenuation. On the otherhand, for $\omega < \omega_p$, k becomes purely imaginary and the wave gets attenuated. Most importantly, the expression for the plasma frequency in the present case has two terms, the first term corresponds to the plasma frequency of a conventional nodal metal whereas the second term which is due to CME, modifies it in a Weyl semimetal. Since the plasma frequency here is the difference between two terms, the second term will necessarily decline its value, even it can make ω_p vanish when both the terms are exactly equal. Further, the plasma frequency ω_p is directly proportional to the chemical potential which is in accordance with the earlier work[44, 45]. Considering the parameters namely, $\mu_0 \sim \text{few}[eV]$, $v_F \sim 10^5[ms^{-1}]$ and others mentioned earlier, ω_p is estimated to be in $[meV]$ range which is much less as compared to the conventional metals(plasma frequency in the conventional metals are in the range of $[eV]$). The effect of chiral anomaly on the plasma mode in both the intrinsic and doped WSM has been recently investigated microscopically in random phase approximation(RPA)[46] and it has been shown that the chiral anomaly leads to unconventional plasmon mode giving rise to frequency in the $[meV]$ range. Thus, our result, eventhough is purely classical, agrees well with that of a microscopic theory if one neglects the logarithmic corrections. The dielectric function as well as the refractive index which is related to the real part of k as, $\sqrt{\epsilon} = n = \frac{ck_R}{\omega}$, their relation to plasma frequency is given as, $\epsilon = n^2 = 1 - \frac{\omega_p^2}{\omega^2}$. Due to the smallness of the plasma frequency in WSM, the frequency dependence of these quantities will behave accordingly depending on whether $\omega < \omega_p$ or $\omega > \omega_p$.

So far, we have discussed the role of finite energy separation of nodes in a Weyl semimetal and its consequences interms of macroscopic electrodynamics. This

leads to CME which provides significant changes in the properties, namely, the frequency dependent skin depth, the capacitive transport, the plasma frequency as well as the refractive index respectively in a non-trivial way as compared to conventional metals. However, recently it has been shown in a two-band lattice model that the Weyl nodes and chirality are not required to obtain CME while they remain crucial for the chiral anomaly[47]. The CME, similar to anomalous Hall effect results directly

from the Berry curvature of the energy bands even when there are no monopole source from the Weyl nodes. The phenomena of CME, thus can be observed in a wider class of materials.

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